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LOGARITHMIC INVENTORY MODEL WITH SHORTAGE FOR DETERIORATING ITEMS

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Abstract: In this paper, we have modeled a business process which starts with shortage of deteriorating items. After a duration managers have freedom to order the stock of assurance of committed customers. There are many products that follow logarithmic demand pattern, so in this paper we incorporate it with the shortage of items at the beginning. A new model is developed to obtain the optimal solution for such type of market situation and have obtained some valuable results. Numerical examples and simulation study is appended along with managerial insights.

Keywords: Inventory, cycle time, optimality, deterioration, shortage, logarithmic demand.

MSC: 90B05, 90B30, 90B50.

1. INTRODUCTION

A business could start with shortages, like advance booking of LPG gas, electricity supply, and pre-public offer of equity share of company before properly functioning it. In the proposed model, we incorporate two objects, where one is logarithmic demand and the other is the business started with shortages. Few items in the market are of high need for people, like sugar, wheat, oil, whose shortage break the customer's faith and arrival pattern. This motivates retailers to order an excessive quantity of units of an item, in spite of deterioration. Therefore, the loss due to damage, decaying, spoilage or due to deterioration can not be negligible. As inventory is defined as decay change, damaged or spoiled items can not be used for their original purposes. Moreover, deterioration is manageable for many items by virtue of modern advanced storage technologies. We have incorporated deterioration factor in the proposed model. Inventory model presents a real life problem (situation) which helps to run the business smoothly. Burwell *et al.* (1997) solved the problem arising in business by providing freight discounts and presented economic lot size model with price-dependent demand.

Shin (1997) and Khedlekar (2012) determined an optimal policy for retail price and lot size under day-term supplier credit. Shukla and Khedlekar (2010*a*) introduced a three-component demand rate for newly launched deteriorating items with two policies based on constant demand rates and after maturing the product in market, it follows linear demand. Matsuyama (2001) presented a general EOQ model considering holding costs, unit purchase costs, and setup costs that are time-dependent and continuous general demand functions. The problem has been solved by dynamic programming so as to find ordering point, ordering quantity, and incurred costs.

Joglekar (2003) used a linear demand function with price sensitiveness and allowed retailers to use a continuous increasing price strategy in an inventory cycle. He derived the retailer's optimal profit by ignoring all the inventory costs. His findings are not restricted to growing market only, which is neither for a stable market nor for a declining market. The research overview Emagharby and Keskinocak (2003) is for determining the dynamic pricing and order level. Teng and Chang (2005) presented an economic production quantity (EPQ) model for deteriorating items when the demand rate depends not only on the on-display stock, but also on the selling price per unit considering market demand. The manipulation in selling price is the best policy for the organization as well as for the customers.

Wen and Chen (2005) suggested a dynamic pricing policy for selling a given stock of identical perishable products over a finite time horizon on the internet. The sale ends either when the entire stock is sold out, or when the deadline is over. Here, the objective of the seller is to find a dynamic pricing policy that maximizes the total expected revenues.

The EOQ model designed by Hou and Lin (2006) reflects how a demand pattern which is price, time, and stock dependent affects the discount in cash. They discussed an EOQ inventory model which takes into account the inflation and time value of money of the stock-dependent selling price. Existence and uniqueness of the optimal solution has not been shown in this article.

Hill (1995) was the first to introduce the ramp type demand rate in inventory model. The ramp type demand is commonly seen when some fresh fruits are brought to the market. In such type of demand, Hill considered increases linearly at the beginning, and then after maturation the demand becomes a constant, a stable stage till the end of the inventory cycle. You (2005) discussed a dynamic inventory policy for product with price and time-dependent demand. He determined jointly the order size and optimal prices when a decision maker had the opportunity to adjust price before the end of sale season. The problem has been solved so to satisfy Kuhn–Tucker's necessary condition.

Lai *et al.* (2006) algebraically approaches the optimal value of cost function rather than the traditional calculus method and modifies the EPQ model earlier presented by Chang (2004), where he considered variable lead time with shortages. Some useful contribution to EPQ models and deterioration are due to Birbil *et al.* (2007) and Hou (2007), Roy (2008), Bhaskaran *et al.* (2010), Khedlekar (2012, 2013), Kumar and Sharma (2012*a*, *b* & *c*), and Yadav (2012). Motivation of present problem is derived due to Wu (2002), Deng (2007), Roy and Chaudhuri (2012) and Shukla *et al.* (2009, 2010*b* & *c*) for consideration shortages at the beginning of a business, and the results are simulated by numerical examples.

2. ASSUMPTIONS AND NOTATIONS

Assume that the demand of a product is $D(t) = a \log(bt)$, (a > 1, b > 1) and shortages accumulated till time t_1 up to level $I_1(t_1)$ and order received to the company by vendor at time t_1 and thus shortage fulfilled and inventory reaches up to level $I_2(t_1)$. The inventory level $I_2(t_1)$ is sufficient to fulfill the demand till time *T*. Our aim is to find the optimal time t_1 , $I_1(t_1)$ and $I_2(t_1)$, which minimize the total inventory cost. Inventory depletion is shown in Fig 1.



Figure: 1 (Inventory depletion for a cycle time)

Following notations bearing the concepts utilized in the discussion are given as bellow:

- D(t): demand of product is $D(t)=a\log(bt)$, where a and b > 1 are positive real values
 - θ : rate of deterioration $0 \le \theta < 1$.
 - c_1 : holding cost unit per unit time,
 - c_2 : shortage cost unit per unit time,
 - c_3 : deterioration cost,
 - T : cycle time,
 - t_{\perp}^* : optimal time for accumulating shortage,
- $C(t^*)$: optimal average inventory cost,
- D_T : total deteriorated units,
- S_T : total shortage units in the system,
- S_C : total shortage cost,
- H_C : total holding cost,
- D_C : total deterioration cost.

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3. MATHEMATICAL MODEL

Suppose that on hand shortages denoted by $I_1(t)$ are accumulated till time t_1 . Management placed the order at time t_1 , which is immediately fulfilled, and thus on hand inventory is $I_2(t)$. After time t_1 inventory depleted due to demand and deterioration, and reduces gradually to zero at time T (see Fig. 1).

$$\frac{d}{dt}I_{1}(t) = -a\log(bt), \text{ where } 0 \le t \le t_{1}, I_{1}(0) = 0, a > 1, b > 1,$$
(1)

$$\frac{d}{dt}I_2(t) + \theta I_2(t) = -a\log(bt), \text{ where } t_1 \le t \le T$$
(2)

Boundary conditions for above two differential equations are $I_1(0) = 0$, $I_2(T) = 0$

On solving equation (1), we get

$$I_{1}(t) = A - \int_{0}^{t} a \log(bu) \, du \text{ , with } I_{1}(0) = 0$$

$$I_{1}(t) = at - at \log(bt) \tag{3}$$

On solving equation (2), we get

$$I_2(t)e^{\theta t} = B - \int_{t_1}^t a \log(bu) du$$
, with $I_2(T) = 0$

Substituting B, obtained from boundary condition $I_2(T) = 0$, in the above equation, we get

$$I_{2}(t) = a_{1} - a\theta Tt \log(bT) - at \log(bt) - a(T-t) + a\theta \left(Tt - \frac{3t^{2}}{4} - \frac{T^{2}}{4}\right)$$
(4)

where
$$a_1 = a \left(T + \frac{\theta T^2}{2} \right) \log(bT)$$

Deteriorated units (D_T) in time $(t_1, T]$ is

$$D_{T} = I_{2}(t_{1}) - \int_{t_{1}}^{T} a \log(bt) dt, \quad 0 \le t \le T$$
(5)

$$= a_1 - a\theta t_1 T \log(bT) - aT \log(bT) + a\theta \left(Tt_1 - \frac{3t_1^2}{4} - \frac{T^2}{4} \right)$$
(6)

Holding cost H_C , over time $(t_1, T]$ will be

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$$H_{C} = c_{1} \int_{t_{1}}^{T} e^{-\theta t} \left[\int_{t}^{T} e^{\theta u} a \log(bu) du \right] dt$$
⁽⁷⁾

$$=c_{1}\left\{ \left(T-t_{1}\right)\left(a_{1}-aT\right)-\frac{a\theta T}{2}\left(T^{2}-t_{1}^{2}\right)\log(bT)-\frac{aT^{2}}{2}\log(bT)+\frac{at_{1}^{2}}{2}\log(bt_{1})\right\}$$
(8)

$$+c_{1}\left\{\frac{3a}{4}\left(T^{2}-t_{1}^{2}\right)+\frac{a\theta}{4}\left(2T^{3}-2Tt_{1}^{2}+t_{1}^{3}-Tt_{1}\right)\right\}$$

Shortages = $I_1(t_1) = at_1 - at_1 \log(bt_1)$, and shortage cost S_c is

$$Sc = \int_0^t I_1(t) dt = \frac{3}{4} a t_1^2 - a t_1^2 \log(bt_1)$$
(9)

Number of units including shortage in business will be Q

$$Q = I_{1}(t_{1}) + I_{2}(t_{1})$$

= $aT \left(\log(bT) - 1 \right) + a\theta T \log(bT) \left(\frac{T}{2} - t_{1} \right) + a\theta \left(Tt_{1} - \frac{3}{4}t_{1}^{2} - \frac{T^{2}}{4} \right)$
+ $2at_{1} \log(bt_{1}) + 2at_{1}$ (11)

Total average inventory cost will be

$$C(t_{1}) = \left(\frac{H_{C} + S_{C} + D_{C}}{T}\right)$$

$$= \frac{1}{T} \left[c_{1} \left\{ (T - t_{1}) (a_{1} - aT) - \frac{a\theta T}{2} (T^{2} - t_{1}^{2}) \log(bT) - \frac{aT^{2}}{2} \log(bT) \right\} \right]$$

$$+ \frac{1}{T} \left[c_{1} \left\{ \frac{3a}{4} (T^{2} - t_{1}^{2}) \frac{a\theta}{4} (2T^{3} - 2Tt_{1}^{2} + t_{1}^{3} - Tt_{1}) + \frac{at_{1}^{2}}{2} \log(bt_{1}) \right\} \right]$$

$$+ \frac{c_{3}}{T} \left[a_{1} - a\theta t_{1}T \log(bT) - aT \log(bT) + a\theta \left[Tt_{1} - \frac{3t_{1}^{2}}{4} - \frac{T^{2}}{4} \right] \right]$$

$$+ \frac{1}{T} \left[ac_{2}t_{1}^{2} \left(\frac{3}{4} \log(bt_{1}) \right) \right]$$

$$\frac{d}{dt} C(t_{1}) = \frac{1}{T} \left\{ \frac{aT - a_{1} + a\theta Tt_{1} \log(bT) + \frac{at_{1}}{2} + at_{1} \log(at_{1})}{-\frac{3at_{1}}{2} + \frac{a\theta}{4} (3t_{1}^{2} - 4Tt_{1} - T^{2})} \right\}$$

$$(13)$$

$$+ \frac{1}{T} \left\{ 2ac_{2}t_{1} \left(\frac{3}{4} - \log(bt_{1}) \right) - a\theta Tc_{3} \log(bT) - \frac{3a\theta c_{3}t_{1}}{2} \right\}$$

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Condition for optimality $\frac{d}{dt}C(t_1) = 0$, we get equation for optimal value of t_1

$$c_{1}\left(aT - a_{1} - \frac{a\theta T^{4}}{4}\right) - a\theta TC_{3}\log(bT) + t_{1}\left\{a\theta c_{1}T\log(bT) - ac_{1} + ac_{1}\log(bt_{1}) - a\theta c_{1}T + ac_{2} + ac_{2}\left(\frac{3}{4} - \log(bt_{1})\right) - \frac{3}{2}a\theta c_{3}\right\}$$
(14)
+ $t_{1}^{2}\left\{\frac{3a\theta c_{1}}{4}\right\} = 0$

Suppose that the optimal value obtained from the above equation is t_1^*

Condition for optimality is

$$\frac{d^2}{dt^2}C(t_1) = \frac{1}{T} \left\{ ac_1 \log(bt_1) - 3ac_2 + \frac{3a\theta c_1}{2}t_1 + ac_1 T\log(bT) - a\theta c_1 T + 2ac_2 \left(\frac{3}{4} - \log(bt_1)\right) - \frac{3a\theta c_3}{2} \right\}$$
(15)

at
$$t_1 = t_1^*, \ \frac{d^2}{dt^2} C(t_1^*) \ge 0$$
 (16)

Thus $C(t_1^*)$ is optimum.

4. NUMERICAL EXAMPLE

To illustrate the model, assume that parameters are a = 20 units, b = 0.2, $c_1 = \$1.4$ per unit, $c_2 = \$2$ per unit, $C_3 = \$2$ per unit, $\theta = 0.01$ and T = 14 days and demand of the product is $D(t) = a \log(bt)$. Under the given parameter values and by equation (5) to (12), we get output parameters: $t_1 = 2.955$ days, optimal quantity Q = 153 units, average holding cost $H_C = \$13.52$ and average total inventory cost $C(t_1^*) = \$228.69$.

5. SENSITIVITY ANALYSIS

In this section, we investigate how the input parameters change significantly the output parameters. We change in one parameter and keeping other parameters invariant. The base data are got accordingly to the numerical example.

Variation in Parameter	b	а	C ₁	<i>C</i> ₂	C3	θ	Т	t _I	TC	Q(T)	$I_2(t_2)$	Holding Cost	Shortage Cost	D_T
Т	0.2	10	1	2	2	0.01	07	6.266	61.56	53	5	19.2000	48.59	0
	0.2	10	1	2	2	0.01	08	6.125	60.11	62	13	70.2400	48.88	0
	0.2	10	1	2	2	0.01	10	5.713	68.47	80	30	281.140	1.000	0
	0.2	10	1	2	2	0.01	12	5.108	89.58	95	45	691.700	49.99	2
	0.2	10	1	2	2	0.01	14	4.234	122.43	101	51	1378.56	49.34	4
θ	0.2	10	1	2	2	0.005	14	4.600	120.99	104	55	1338.05	49.82	2
	0.2	10	1	2	2	0.01	14	4.234	122.43	101	52	1378.56	49.35	4
	0.2	10	1	2	2	0.02	14	3.406	125.32	76	29	1475.17	47.07	9
C3	0.2	10	1	2	2	0.01	14	4.234	122.43	101	51	1378.56	49.34	4
	0.2	10	1.4	9	3	0.01	14	5.855	243.66	115	66	1574.80	49.35	2
	0.2	10	1.4	9	4	0.01	14	5.841	243.96	115	66	1577.92	49.37	2
	0.2	10	1.4	9	5	0.01	14	5.826	244.26	115	66	1581.28	49.40	2
	0.2	10	1.4	9	6	0.01	14	5.812	244.55	115	66	1584.41	49.42	3
	0.2	10	1.4	9	7	0.01	14	5.798	244.85	115	66	1587.54	49.44	3
	0.2	10	1.4	9	8	0.01	14	5.781	245.16	115	66	1591.32	49.46	3
C2	0.2	10	1.4	3	2	0.01	14	4.465	171.68	104	54	1880.65	49.68	4
	0.2	10	1.4	4	2	0.01	14	5.048	179.63	110	60	1753.92	50.00	3
	0.2	10	1.4	5	2	0.01	14	5.361	190.61	113	63	1684.85	49.89	3
	0.2	10	1.4	6	2	0.01	14	5.559	202.94	114	64	1640.87	49.73	3
	0.2	10	1.4	7	2	0.01	14	5.695	216.02	115	65	1610.55	49.58	3
	0.2	10	1.4	8	2	0.01	14	5.794	229.55	115	66	1588.45	49.44	2
	0.2	10	1.4	9	2	0.01	14	5.870	243.36	115	66	1571.44	49.33	2
<i>c</i> ₁	0.2	10	0.8	2	2	0.01	14	4.740	100.58	107	57	1040.70	49.92	3
	0.2	10	0.9	2	2	0.01	14	4.496	111.26	104	55	1204.71	49.71	4
	0.2	10	1.2	2	2	0.01	14	3.648	146.45	91	44	1759.14	47.92	4
	0.2	10	1.4	2	2	0.01	14	2.957	172.96	77	32	2190.11	45.02	5
	0.2	10	1.5	2	2	0.01	14	2.520	187.58	65	23	2435.09	42.38	5
а	0.2	15	1.4	2	2	0.01	14	2.957	200.82	115	47	2464.51	67.53	7
	0.2	20	1.4	2	2	0.01	14	2.955	228.69	153	63	2739.45	90.03	9
	0.2	25	1.4	2	2	0.01	14	2.954	256.56	191	79	3014.24	112.5	11
	0.2	30	1.4	2	2	0.01	14	2.953	284.43	230	95	3289.10	135.0	13

 Table 1. Sensitivity of different parameters





Total inventory cost increases as time cycle T increases (see fig 2) and is followed by economic order quantity (table1). Both economic order quantity and incurred cost increase as shortage cost increases (see fig 3 and table 1), but this increment is nonlinear. For smaller c_2 , the increment in Q is faster and saturates latter. Total deterioration

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cost also increases lineary as c_3 increases. Thus deterioration cost is negatively affected by c_3 (see fig 4 and 5). Managers need to be aware of deterioration cost and holding cost, and keep it as low as possible in order to keep lower average cost. High initial demand parameter (*a*) increases EOQ, and total average cost both (table 1), but optimal time remains unchanged. From table 1, it is observed that the optimal time is highly sensitive on deterioration and holding cost.

6. CONCLUSION

A solution of proposed inventory problem is obtained for a business cycle which starts with shortage and follows logarithmic demand. Simulation study reveals that suggested model is highly sensitive on the shortage cost, so inventory managers should negotiate this with retailers intelligently as to keep the cost lower. It is found that logarithmic demand is less dependent on time, and high initial demand increases EOQ correspondingly. Mostly output are less dependent on cycle time so, managers are allowed to keep longer cycle time.

The shortage cost and EOQ have non-linear relationship. For lower shortage cost, increment rate in EOQ is relatively high. This model can further be extended to varying deterioration, ramp type demand with finite rate of replenishment. One could also formulate the similar model in the fuzzy environment.

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