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DISCUSSION ON FUZZY DECISION MAKING BASED ON FUZZY NUMBER AND COMPOSITIONAL RULE OF INFERENCE

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Abstract: This paper provides an improved decision making approach based on fuzzy numbers and the compositional rule of inference by Yao and Yao (2001). They claimed to have created a new method that combines statistical methods and fuzzy theory for medical diagnosis. Currently, numerous papers have cited that work. In this study, we show that if we follow their matrix multiplication operation approach, we will obtain the same result as the original method proposed by Klir and Yuan (1995). Owing to a well-known property of (row) stochastic matrices, if the multiplication is closed, the fuzzy and defuzzy procedure of Yao and Yao (2001) is redundant. Therefore, we advise researchers to think twice before applying this approach to medical diagnosis.

Keywords: Fuzzy decision making, Fuzzy number, Medical diagnosis, Interval estimate.

MSC: 62C86, 90B50.

1. INTRODUCTION

Diagnosing a medical condition is a very complex procedure. The diagnosis can be considered a tag conferred by the doctor to describe the condition of a patient. Observing the symptoms of the patient and his or her medical information, the doctor synthesizes the information by noting the symptoms, performing a clinical examination of the patient, and analyzing the patientös medical history. In order to handle the uncertainty that comes from viewing a patientös symptoms, the doctor deals with vagueness in choosing the diagnostic tag to insure the proper healing process.

The fuzzy set framework has been utilized in several different approaches to model the diagnostic process. The approach formulated by Sanchez [27] in 1979 adopted the compositional rule of inference by Zadeh [33] as an inference mechanism. It accepted fuzzy descriptions of a patientôs symptoms and inferred fuzzy descriptions of the patientôs diseases by means of the fuzzy relationships. Furthermore, several researchers followed Sanchezôs [27] approach to extend fuzzy inference to medical engineering, notable examples being Pavlica and Petrovacki [24], Steimann and Adlassnig [28], Innocent and John [15], Palma et al. [23], Seising [29], Quteishat and Lim [25], and Ahn et al. [1].

In addition, Yao and Yao [32] have solved medical diagnostic problems based on fuzzy numbers and the compositional rule of inference. However, the authors have found that Yao and Yaoôs [32] theoretical derivations and analytical results are questionable. If we follow their approach with the matrix multiplication operation, the result will be identical to that of the original method proposed by Klir and Yuan [17]. Owing to a well-known property of (row) stochastic matrices, if the multiplication is closed, then the normalization procedure of Yao and Yao [32] is redundant.

In addition, currently 27 papers have cited Yao and Yao [32] (such as Fenza et al. [7]; Huang [13]; Hung [14]; Lin and Lee [19]; Fenza et al. [6]; Mahmoodabadi et al. [21]; Al-Hawari et al. [4]; Ahn et al. [1]; Mahmoodabadi et al. [20]; Zeng [34]; Pal et al. [22]; Goyal et al. [8]; Ahn et al. [3]; Zeng et al. [35]; Rakus-Andersson [26]; Hong et al. [12]; Fang and Huang [5]; Yao and Yu [31]; He and Jennings [9]; Levin and Sokolova [18]; He et al. [10]). However, they have all failed to point out that the work of Yao and Yao [32] is questionable. Consequently, this study aims to prove that their method is redundant.

The remainder of this study is organized as follows: Section 2 reviews the mathematical formulation of Yao and Yaoôs [32] research and provides an example to illustrate their questionable results. Section 3 is the proof demonstrating that the normalization procedure of Yao and Yao [32] is redundant. Finally, we draw a conclusion in section 4.

2. REIVEW OF THEIR RESULTS

Yao and Yao [32] assumed $\tilde{A}=(a,b,c)$ to be the triangular fuzzy number with membership function

$$\mu_{\tilde{A}}(x)=\begin{cases}0 & x<a\\ (x-a)/(b-a) & a\le x\le b\\ (c-x)/(c-b) & b\le x\le c\\ 0 & x>c.\end{cases} \tag{1}$$

Its centroid is

$$M_{\tilde{A}} = \frac{\int_{-\infty}^{\infty} x\mu_{\tilde{A}}(x)\,dx}{\int_{-\infty}^{\infty} \mu_{\tilde{A}}(x)\,dx} = \frac{1}{3}(a+b+c). \tag{2}$$

From Yao and Wu [30], the α–cut of $\tilde{A}$ is $[A_L(\alpha), A_R(\alpha)]$, for $0 \le \alpha \le 1$, with $A_L(\alpha) = a + (b-a)\alpha$, and $A_R(\alpha) = c - (c-b)\alpha$.

According to Yao and Wu [30], Definition 2.5, the signed distance of two fuzzy sets $\tilde{A} = (a,b,c)$ and $\tilde{B} = (p,q,r)$ is denoted as

$$d(\tilde{A},\tilde{B}) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha) - B_L(\alpha) - B_R(\alpha)]d\alpha = \frac{1}{4}(2b + a + c - 2q - p - r). \tag{3}$$

Hence, the sign distance of $\tilde{A} = (a,b,c)$ and $\tilde{0}_1 = (0,0,0)$ is expressed as

$$d(\tilde{A},\tilde{0}_1) = \frac{1}{4}(2b + a + c). \tag{4}$$

There are two compositional rules of inference in Yao and Yao [32]: (a) max-min operation, and (b) matrix multiplication, subsequently trimmed by unity. For example, if we assume that $\tilde{A} = (a_1, a_2)$ and $\tilde{B} = (b_1, b_2)$, then by the max-min operation,

$$\tilde{A} \circ \tilde{B} = (a_1 \wedge b_1) \vee (a_2 \wedge b_2) \tag{5}$$

where $\wedge$ means the minimum and $\vee$ means the maximum.

The other operation in Yao and Yao [32] is denoted

$$\tilde{A} \circ \tilde{B} = \min\left\{1, \sum_{i=1}^{2} a_i b_i\right\}, \tag{6}$$

as the matrix multiplication of a row matrix and a column matrix (or inner product of two vectors), subsequently cut by unity.

In this study, we will only consider the compositional rules of inference by the matrix multiplication subsequently trimmed by unity.

$S = \{S_1, S_2, ..., S_n\}$ is the set of symptoms. $D = \{d_1, d_2, ..., d_m\}$ is the set of diseases. $P = \{P_1, P_2, ..., P_k\}$ is the set of patients. The fuzzy relation between patient and symptoms is expressed as $\tilde{Q}$, and the fuzzy relation between symptoms and diseases is denoted by $\tilde{R}$.

For symptom S_i and disease d_j , the proportion of the patient population is assumed to be P_{ij} . The point estimation of P_{ij} is r_{ij} by a comprehensive study of sample size, *N*. However, the point estimation is unable to present the probability error, so Yao and Yao [32] suggested the use interval estimation as follows. From statistical theory, they knew that the $(1-\alpha)100\%$ confidence interval of population proportion *p* is

$$\left[\hat{p}-t_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+t_\alpha\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] \tag{7}$$

where $\hat{p}$ is the point estimate of p, n is the sample size, and t_α is the number that satisfies $\Pr\left(\left|N(0,1)\right| \ge t_\alpha\right)=\alpha$ with $N(0,1)$ as the standard normal distribution.

Hence, Yao and Yao [32] converted a crisp estimation r_{ij} for symptom S_i and disease d_j into an interval estimation of the 99% confidence interval as $[r_{ij}^{(1)}, r_{ij}^{(2)}]$ with

$$r_{ij}^{(1)} = r_{ij} - 2.575\sqrt{\frac{r_{ij}\left(1-r_{ij}\right)}{N_{ij}}}, \text{ and}$$

$$r_{ij}^{(2)} = r_{ij} + 2.575\sqrt{\frac{r_{ij}\left(1-r_{ij}\right)}{N_{ij}}}$$

for $i=1,\dots,n$, and $j=1,\dots,m$, where N_{ij} is the number of patients who have symptom S_i in the disease d_j, and then $N_i=\sum_{j=1}^{m} N_{ij}$ and $r_{ij}=\frac{N_{ij}}{N_i}$. Because $\Pr=\left(\left|N(0,1)\right| \ge 2.575\right)$ =0.01, then for the 99% confidence interval, $t_{0.01}=2.575$.

Yao and Yao [32] used $\tilde{r}_{ij}$ to denote the fuzzy number $\left(r_{ij}^{(1)}, r_{ij}, r_{ij}^{(2)}\right)$ to present a triangular fuzzy number. Hence, they generalized the crisp environment of point estimation,

$$\tilde{R}=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix} \tag{8}$$

into a fuzzy environment of interval estimation,

$$\tilde{R}^*=\begin{bmatrix} \tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1m} \\ \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{r}_{n1} & \tilde{r}_{n2} & \cdots & \tilde{r}_{nm} \end{bmatrix} \tag{9}$$

with $\tilde{r}_{ij}=\left(r_{ij}^{(1)}, r_{ij}, r_{ij}^{(2)}\right)$, for $i=1,\dots,n$, and $j=1,\dots,m$, for the relation between symptom and disease. Therefore, the composition operation is also extended from the crisp environment into the fuzzy environment. Under the crisp condition, Yao and Yao [32] applied the composition operation of equation (6) to find the relation between patient and disease to obtain the result as follows

$$\tilde{Q}\circ\tilde{R}=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{k1} & a_{k2} & \cdots & a_{kn}\end{bmatrix}\circ\begin{bmatrix}r_{11} & r_{12} & \cdots & r_{1m}\\ r_{21} & r_{22} & \cdots & r_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ r_{n1} & r_{n2} & \cdots & r_{nm}\end{bmatrix}=\begin{bmatrix}t_{11} & t_{12} & \cdots & t_{1m}\\ t_{21} & t_{22} & \cdots & t_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ t_{k1} & t_{k2} & \cdots & t_{km}\end{bmatrix}, \tag{10}$$

with

$$t_{ij}=\min\left\{1,\sum_{k=1}^{n}a_{ik}r_{kj}\right\}. \tag{11}$$

On the other hand, for the fuzzy condition, Yao and Yao [32] used the fuzzy arithmetic operation of Kaufmann and Gupta [16] and Zimmermann [36] to derive the relation between patient and disease, that

$$\tilde{Q}\circ\tilde{R}=\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{k1} & a_{k2} & \cdots & a_{kn}\end{bmatrix}\circ\begin{bmatrix}\tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1m}\\ \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{r}_{n1} & \tilde{r}_{n2} & \cdots & \tilde{r}_{nm}\end{bmatrix}=\begin{bmatrix}\tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1m}\\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{t}_{k1} & \tilde{t}_{k2} & \cdots & \tilde{t}_{km}\end{bmatrix}, \tag{12}$$

with $\tilde{t}_{ij}=\left(\sum_{k=1}^{n}a_{ik}r_{kj}^{(1)},\sum_{k=1}^{n}a_{ik}r_{kj},\sum_{k=1}^{n}a_{ik}r_{kj}^{(2)}\right)$, and then they used the centroid to defuzzify it and obtain

$$\begin{aligned}M_{\tilde{t}_{ij}}&=\frac{1}{3}\left(\sum_{k=1}^{n}a_{ik}r_{kj}^{(1)}+\sum_{k=1}^{n}a_{ik}r_{kj}+\sum_{k=1}^{n}a_{ik}r_{kj}^{(2)}\right)\\ &=\sum_{k=1}^{n}a_{ik}\,\frac{1}{3}\left(r_{kj}^{(1)}+r_{kj}+r_{kj}^{(2)}\right)=\sum_{k=1}^{n}a_{ik}r_{kj}.\end{aligned} \tag{13}$$

They also used the signed distance [30] to compute

$$d(\tilde{t}_{ij},\tilde{0}_1)=\frac{1}{4}\left(\sum_{k=1}^{n}a_{ik}r_{kj}^{(1)}+2\sum_{k=1}^{n}a_{ik}r_{kj}+\sum_{k=1}^{n}a_{ik}r_{kj}^{(2)}\right)=M_{\tilde{t}_{ij}}. \tag{14}$$

Yao and Yao [32] compared equations (11) and (13) to mentioned that $M_{\tilde{t}_{ij}}\geq t_{ij}$ Moreover, if $M_{\tilde{t}_{ij}}\leq 1$, then $M_{\tilde{t}_{ij}}=t_{ij}$. For later discussion, we wrote the relation between patient P_i and disease d_j for $j=1,\ldots,m$ under the crisp condition,

$$\left(t_{i1}=\min\left\{1,\sum_{k=1}^{n}a_{ik}r_{k1}\right\},\ldots,t_{im}=\min\left\{1,\sum_{k=1}^{n}a_{ik}r_{km}\right\}\right). \tag{15}$$

After normalization, it yields the probability distribution for patient P_i corresponding to diseases d_j for $j = 1,...,m,$ under the crisp condition,

$$\left(\frac{\min\left\{1, \sum_{k=1}^{n} a_{ik} r_{k1}\right\}}{\sum_{j=1}^{m} \min\left\{1, \sum_{k=1}^{n} a_{ik} r_{kj}\right\}}, \ldots, \frac{\min\left\{1, \sum_{k=1}^{n} a_{ik} r_{km}\right\}}{\sum_{j=1}^{m} \min\left\{1, \sum_{k=1}^{n} a_{ik} r_{kj}\right\}} \right). \tag{16}$$

On the other hand, the relation between patient P_i and disease d_j for $j = 1,...,m,$ under the fuzzy condition, is

$$\left(M_{\tilde{t}_{ij}} = \sum_{k=1}^{n} a_{ik} r_{k1}, \ldots, M_{\tilde{t}_{im}} = \sum_{k=1}^{n} a_{ik} r_{km} \right). \tag{17}$$

After normalization, it yields the probability distribution for patient P_i corresponding to diseases d_j for $j = 1,...,m,$ under the fuzzy condition, as follows:

$$\left(\frac{\sum_{k=1}^{n} a_{ik} r_{k1}}{\sum_{j=1}^{m} \sum_{k=1}^{n} a_{ik} r_{kj}}, \ldots, \frac{\sum_{k=1}^{n} a_{ik} r_{km}}{\sum_{j=1}^{m} \sum_{k=1}^{n} a_{ik} r_{kj}} \right). \tag{18}$$

In general, the probability distributions derived from the crisp environment and by the fuzzy environment are not the same. Yao and Yao [32] even provided us with an example to demonstrate that they are sometimes equal.

However, Yao and Yao [32] did not provide further discussion on how to handle the difference between the crisp environment and the fuzzy environment. The purpose of this paper is to prepare a reasonable patchwork to unify these two results. Moreover, after our improvement, it can be seen that the fuzzy procedure proposed by Yao and Yao [32] for the relation between symptom and disease and then the defuzzify process by the centroid or the signed distance [30] will imply the same results as the crisp case. Hence, Yao and Yaoȏs fuzzy approach appears to be redundant.

For an easy explanation, we quote a numerical example in Yao and Yao [32] to explain their procedure in detail.

Example. We reconsidered the example 1 of Yao and Yao [32]. Set n = 3; m = 4; k = 2, and S = {S_1(headache), S_2 (fever), S_3(phlegm)}, D = {d_1(cold), d_2(pulmonary tuberculosis), d_3(pertussis), d_4(pneumonia)} and P = {P_1, P_2}. The diagnosis of two patients by a physician can derive the following $\tilde{Q}$, and based on a comprehensive survey of a 50-patient study, they obtain the following $\tilde{R}$ for the crisp relation between symptom and disease:

$$\tilde{Q} = \begin{array}{c} \\ P_1 \\ P_2 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{pmatrix} 0.9 & 0.9 & 0.9 \\ 0.5 & 0.2 & 0 \end{pmatrix} \end{array},$$

$$\tilde{R} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{cccc} d_1 & d_2 & d_3 & d_4 \end{array} \\ \begin{pmatrix} 0.4 & 0.2 & 0.1 & 0.3 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.2 \end{pmatrix} \end{array}. \tag{19}$$

Then the traditional matrix multiplication implies that

$$\left[\tilde{Q}\right]\left[\tilde{R}\right] = \begin{bmatrix} 0.72 & 1.08 & 0.36 & 0.54 \\ 0.26 & 0.18 & 0.09 & 0.17 \end{bmatrix}. \tag{20}$$

According the result of equation (20) and fuzzy operation in equation (11), then

$$\tilde{T} = \tilde{Q} \circ \tilde{R} = \begin{bmatrix} 0.72 & 1 & 0.36 & 0.54 \\ 0.26 & 0.18 & 0.09 & 0.17 \end{bmatrix}. \tag{21}$$

After the normalization of $\tilde{T}$, Yao and Yao [32] got

$$\begin{bmatrix} \frac{0.72}{2.62} & \frac{1}{2.62} & \frac{0.36}{2.62} & \frac{0.54}{2.62} \\ \frac{0.26}{0.7} & \frac{0.18}{0.7} & \frac{0.09}{0.7} & \frac{0.17}{0.7} \end{bmatrix} = \begin{bmatrix} 0.2748 & 0.3817 & 0.1374 & 0.2061 \\ 0.3714 & 0.2571 & 0.1286 & 0.2429 \end{bmatrix}. \tag{22}$$

On the other hand, for the fuzzy approach, by the centroid or the signed distance [30], Yao and Yao [32] imply that

$$\left[M_{\tilde{t}_{ij}}\right] = \left[d\left(\tilde{t}_{ij}, \tilde{0}_1\right)\right] = \begin{bmatrix} 0.72 & 1.08 & 0.36 & 0.54 \\ 0.26 & 0.18 & 0.09 & 0.17 \end{bmatrix}, \tag{23}$$

and then the probability distribution is derived as follows:

$$\begin{bmatrix} \frac{0.72}{2.7} & \frac{1.08}{2.7} & \frac{0.36}{2.7} & \frac{0.54}{2.7} \\ \frac{0.26}{0.7} & \frac{0.18}{0.7} & \frac{0.09}{0.7} & \frac{0.17}{0.7} \end{bmatrix}$$

$$= \begin{bmatrix} 0.2667 & 0.4000 & 0.1333 & 0.2000 \\ 0.3714 & 0.2571 & 0.1286 & 0.2429 \end{bmatrix}. \tag{24}$$

If a physician were to record the result of his examination of patient P_2 from the two approaches, the crisp environment and the fuzzy environment, then the record would have the same probability distribution. For a patient P_1, the probability distributions are different, as demonstrated in the first row of equations (22) and (24).

Yao and Yao [32] are aware that the probability distributions are sometimes different and sometimes the same for the crisp case and the fuzzy case. They did not provide further study to state the cause of such differences.

The main topic of this paper is to provide a reasonable patchwork to amend the differences for the different probability distributions of a patient P_1.

3. OUR REVISIONS

In this section, we will demonstrate that under a reasonable modification of (a) the crisp point estimation based on the operation of equation (12), and (b) Yao and Yaoȏs new interval estimation defuzzified by the centroid or signed distance [30] will derive the same probability distribution. It will reveal that Yao and Yao [32] developed an unnecessary fuzzification.

According to Horn and Johnson [11], we quote the following well-known results in linear algebra. A nonnegative matrix $A \in M_n$ with the property that all its row sums are 1 is said to be a (row) stochastic matrix because each row may be thought of as a discrete probability distribution on a sample space with *n* points (page 526). The set of $n \times n$ stochastic matrices is denoted as M_n to constitute a semigroup under matrix multiplication; that is, if $A, B \in M_n$ are stochastic, then AB is stochastic (page 529).

To be more precise, if there are three $m \times n$ matrices, $A = (a_{st})_{m\times n}$, $B = (b_{tu})_{n\times k}$ and $C = AB = (c_{su})_{m\times k}$ that satisfy the row sum of A or B being the unity that is $\sum_{t=1}^{n} a_{st} = 1$ for $s = 1,2,...,m$ and $\sum_{u=1}^{k} b_{tu} = 1$ for $t = 1,2,...,n$, then $\sum_{u=1}^{k} c_{su} = 1$ for $s = 1,2,...,m$.

For completeness, we provide the following proof:

$$\sum_{u=1}^{k} c_{su} = \sum_{u=1}^{k}\sum_{t=1}^{n} a_{st}b_{tu} = \sum_{t=1}^{n}\sum_{u=1}^{k} a_{st}b_{tu}$$

$$= \sum_{t=1}^{n} a_{st}\sum_{u=1}^{k} b_{tu} = \sum_{t=1}^{n} a_{st} = 1. \tag{25}$$

Since the finite sum can be arbitrarily rearranged, $\sum_{u=1}^{k} b_{tu} = 1$ and $\sum_{t=1}^{n} a_{st} = 1$, we write our results in the next theorem for later use.

Theorem 1. *If* $A = (a_{st})_{m\times n}$ *and* $B = (b_{tu})_{n\times k}$ *are two stochastic matrices, and we assume that* $C = AB = (c_{su})_{m\times k}$*, then* $c_{su} \le 1$ *for* $s = 1,2,...,m$ *and* $u = 1,...,k$.

Proof of Theorem 1. We know that $\sum_{u=1}^{k} c_{su} = 1$ and $c_{su} \ge 0$ for $s = 1,2,...,m$ and $u = 1,...,k$ such that it yields $c_{su} \le 1$.

The purpose of decision making for medical diagnosis is to provide a probability distribution to help physicians determine what disease is affecting a patient. Hence, the final result of the probability distribution in analytical representation in equations (16) and (18), in the numerical example of equations (22) and (24), that satisfies the row sum is equal to one. It reveals that the last step is normalization to make the row sum equal to one.

Based on the above observation, if we record the relation between a patient P_i and symptoms $S_1,\ldots,S_n$ in the early stages, we may at that time normalize the data such that $\tilde{Q}$ and $\tilde{R}$ are both stochastic matrices. Then we can apply our Theorem 1 to imply that

$$\sum_{j=1}^{m} M_{\tilde{t}_{ij}} = 1 \tag{26}$$

and then $0 \le M_{\tilde{t}_{ij}} \le 1$ to obtain that

$$M_{\tilde{t}_{ij}} = t_{ij} = \sum_{k=1}^{n} a_{ik} r_{kj} \,. \tag{27}$$

After our improvement of the normalization of the relation between patient and symptoms, the probability distribution under the fuzzy environment is the same as that under the crisp environment. Therefore, the interval estimation proposed by Yao and Yao [32] to generalize from the crisp condition into the fuzzy condition and then defuzzify it by the centroid and signed distance [30] is a redundant procedure. Moreover, owing to equation (26), the row sum of one patient corresponding to all disease is already one, so the last normalization is unnecessary. We summarize our findings in the next theorem.

Theorem 2. *Under the operation of matrix multiplication and bounded by one, if we normalize the relation of patient and symptoms, then the probability distribution derived by the fuzzy condition is the same as that derived by the crisp condition.*

For completeness, we demonstrate our revision to the previous example, where the row sum of $\tilde{Q}$ is normalized.

$$\tilde{Q} = \begin{array}{c} \\ P_1 \\ P_2 \end{array} \begin{array}{c} \begin{array}{ccc} S_1 & S_2 & S_3 \end{array} \\ \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 5/7 & 2/7 & 0/7 \end{pmatrix} \end{array},$$

$$\tilde{R} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{cccc} d_1 & d_2 & d_3 & d_4 \end{array} \\ \begin{pmatrix} 0.4 & 0.2 & 0.1 & 0.3 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.1 & 0.2 \end{pmatrix} \end{array}. \tag{28}$$

Then the traditional matrix multiplication implies that

$$\left[\tilde{Q}\right]\left[\tilde{R}\right]=\begin{bmatrix} 8/30 & 12/30 & 4/30 & 6/30 \\ 26/70 & 18/70 & 9/70 & 17/70 \end{bmatrix}. \tag{29}$$

Since the row sum in equation (29) is already one, as we proved in Theorem 1, the final normalization of equations (22) and (24) is unnecessary. It is already the probability distribution for the crisp case and the fuzzy case. Hence, after our revision, we found a way to unify the crisp environment and the fuzzy environment. Meanwhile, our patchwork also simplifies the solution procedure.

A similar revision can be rendered for medical diagnosis with patients, symptoms, characteristics, and diseases. Under our revision to normalize the row sum of a patient to symptoms, then the probability distributions for the crisp case and for the fuzzy case will produce the same results. This shows that the fuzzy generalization proposed by Yao and Yao [32] will derive the same results as Klir and Yuan [17] for the crisp environment.

4. CONCLUSION

Yao and Yaoôs method [32] claimed to have created a new method that combined statistical methods and fuzzy theory for medical diagnosis. However, this paper has shown that when the compositional rule of inference is the matrix multiplication and bounded by one, their method is redundant. Consequently, we advise researchers to exercise caution when applying a similar approach to medical diagnosis. Furthermore, we hope that this finding is a contribution to medical science.

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