Yugoslav Journal of Operations Research
25 (2015), Number 3, 445-456
DOI: 10.2298/YJOR131219031E

A PROCEDURE FOR DISTRIBUTING RECRUITS IN MANPOWER SYSTEMS

Virtue U. EKHOSUEHI

Department of Mathematics, University of Benin, P.M.B. 1154, Benin City, Nigeria
virtue.ekhosuehi@uniben.edu

Augustine A. OSAGIEDE

Department of Mathematics, University of Benin, P.M.B. 1154, Benin City, Nigeria
augustine.osagiede@uniben.edu

Wilfred A. IGUODALA

Academic Planning Division, University of Benin, P.M.B. 1154, Benin City, Nigeria
wilfred.iguodala@uniben.edu

Received: December 2013 / Accepted: September 2014

Abstract: In this paper, we treat the following problem: Given a stable Gani-type person-flow model and assuming no negative recruitment, what recruitment distribution at the $n-$step is capable of generating a staff-mix that closely follows the desired structure? We relate this problem to the challenge of universities in Nigeria towards attaining the desired academic staff-mix by rank specified by the National Universities Commission (NUC). We formulate a population-dynamic model consisting of aggregate-fractional flow balance equations within a discrete-time Markov chain framework for the system. We use MATLAB as a convenient platform to solve the system of equations. The utility of the model is illustrated by means of academic staff flows in a university-faculty setting in Nigeria.

Keywords: Gani-type Person-flow Model; Manpower System; Markov Chain; National Universities Commission; Recruitment Distribution.

MSC: 60J20, 91D35.

1. INTRODUCTION

The setting we consider is a manpower system stratified into various categories (states), where negative recruitment is not allowed and a desired staff-mix (structure) is to be attained via a recruitment policy implemented at the $n-$step. We formulate a population-dynamic model consisting of aggregate-fractional flow balance equations within a discrete-time Markov chain framework for the system. We adopt the convention that: $t+n$, $n \in Z_+ = \{1, 2, \cdots\}$, is the $n-$step period starting from the initial period t. The period is a discrete-time scale $\{0, 1, 2, \ \ldots, T\}$. We assume that the number of recruits into the multi-grade manpower system in period $t+n$ is decided by the administrative authority of the system. We use the term 'grade' to mean the status of an individual in a manpower system, and a 'category' to mean the aggregation of grades. For example, in the university system, the grades, denoted as i, are $i=1$ for Graduate Assistant, $i=2$ for Assistant Lecturer, $i=3$ for Lecturer II, $i=4$ for Lecturer I, $i=5$ for Senior Lecturer, $i=6$ for Reader/Associate Professor and $i=7$ for Professor. Our application is focused on the departments of a faculty in the University of Benin, Nigeria. The university system in Nigeria is regulated by the National Universities Commission (NUC). The commission provides guidelines for program evaluation in the university system. Among the guidelines is the academic staff-mix by rank. The staff-mix by rank is that the existing staff structure for academic staff should closely follow the structure 20:35:45 for Professors/Readers: Senior Lecturers: Lecturer I and below (excluding the position of Graduate Assistant), respectively [14]. We perform all computations in the MATLAB environment. The MATLAB program is our choice for this study because it is very flexible and well-suited for matrix-vector algebra, and it contains a library of predefined functions [2, 10].

The Markov chain formulation is a common and attractive approach to modeling a $k-$graded manpower system [1, 4, 7, 13, 16]. The evolution of the manpower system is examined by studying either the structure,

$$\mathbf{n}(t) = [n_1(t) \quad n_2(t) \quad \cdots \quad n_k(t)]',$$

or the relative structure,

$$\mathbf{x}(t) = \left(\sum_{i=1}^{k} n_i(t) \right)^{-1} \mathbf{n}(t)$$

[8, 9, 11, 22].

Among the Markov chain formulations, the Gani-type model [7] has received considerable attention [1, 6, 17]. Specifically, the Gani-type person-flow model, where $\mathbf{n}(t+1) = \alpha \mathbf{n}(t)$ for all $t = 0, 1, 2, \cdots$ and α being the stable growth factor, has been analyzed [6, 17]. New entrants into the system are allocated to a grade i according to a recruitment distribution $\{r_i\}$ with $r_i \geq 0$ and $\sum_{i=1}^{k} r_i = 1$.

Several aspects of the Markov system have been studied in the literature. These include: the behavior of the expectations, variances and covariances of the state sizes [19, 20], the attainability and maintainability of structures [15, 18, 21] and the size order of the state vector [12].

Our study may be seen as an aspect of control in manpower systems. Davies [3] and Bartholomew et al. [1] had earlier provided control strategies for manpower systems. However, the possibility of obtaining negative entries in the recruitment distribution $\{r_i\}$ by the use of the strategies was not resolved. Instead, the negative entries were interpreted as either retrenchment or redundancy. In this study, we circumvent the possibility of obtaining negative entries in the recruitment distribution by redefining the distribution in such fashion that negative recruitment is not allowed. Thus, the trauma associated with negative recruitment is evaded.

2. METHODOLOGY

Let S be a set with labels corresponding to any categorization of interest. For instance, in the NUC categorization, we have $S=\{1,2,3\}$ in an ascending order of categories, as: Lecturer I and below (excluding the position of Graduate Assistant), Senior Lecturers, Professors/Readers. A staff in the system can belong to only one of the categories in S. We use the notation $\mathbf{D}$ to represent an $\#(S)\times 1$ vector of the desired staff-mix, where $\#(S)$ is the cardinality of S. Data for graded manpower systems are normally available as stocks for each grade or flows between grades. We assume that the flows follow a natural order, i.e., promotion is from one grade to the next higher grade, no demotion and no double promotion. Let $0\notin S$ denote the environment outside the manpower system. Suppose the grades $i=\tau$ to ℓ are aggregated into a single category, $\nu\in S$. Then we obtain the time invariant aggregate-fractional flow rates between the categories, denoted as $\hat{p}_{\nu\varsigma}$, for the flows: $\nu\to\nu$ and $\nu\to\nu+1,\ \nu,\nu+1\in S$, as

$$\hat{p}_{\nu\nu}=\frac{\sum_{t=0}^{T}\left(\sum_{i=\tau}^{\ell}n_{i,i}^{(\nu)}(t)+\sum_{i=\tau}^{\ell-1}n_{i,i+1}^{(\nu)}(t)\right)}{\sum_{t=0}^{T}\left(\sum_{i=\tau}^{\ell}n_{i,i}^{(\nu)}(t)+\sum_{i=\tau}^{\ell-1}n_{i,i+1}^{(\nu)}(t)+n_{\ell,\ell+1}^{(\nu)}(t)+\sum_{i=\tau}^{\ell}n_{i,0}^{(\nu)}(t)\right)} \tag{1}$$

and

$$\hat{p}_{\nu,\nu+1}=\frac{\sum_{t=0}^{T}n_{\ell,\ell+1}^{(\nu)}(t)}{\sum_{t=0}^{T}\left(\sum_{i=\tau}^{\ell}n_{i,i}^{(\nu)}(t)+\sum_{i=\tau}^{\ell-1}n_{i,i+1}^{(\nu)}(t)+n_{\ell,\ell+1}^{(\nu)}(t)+\sum_{i=\tau}^{\ell}n_{i,0}^{(\nu)}(t)\right)} \tag{2}$$

provided $n_{\ell,\ell+1}^{(\nu)}(t)$ exists, where $n_{\bullet}^{(\nu)}(t)$ is the number of staff members in the grades τ to ℓ aggregated into a category ν and $n_{ij}^{\bullet}(t)$ is the number of individuals moving from grade i to grade j in period t. The hat denotes an estimate. The movements between the categories are governed by the sub-stochastic matrix, $\mathbf{P} = (\hat{p}_{\nu\varsigma})$. The use of time invariant aggregate-fractional flow rates is necessary to ease the computational agony of obtaining the multi-step transition matrices.

The total stock of the system at period t is

$$N(t) = \sum_{\nu=1}^{\#(S)} \left(\sum_{i=\tau}^{\ell} n_{i,i}^{(\nu)}(t) + \sum_{i=\tau}^{\ell-1} n_{i,i+1}^{(\nu)}(t) + n_{\ell,\ell+1}^{(\nu)}(t) + \sum_{i=\tau}^{\ell} n_{i,0}^{(\nu)}(t) \right) \tag{3}$$

We consider a stable Gani-type person-flow model wherein the total stock $N(t+1)$ satisfies

$$N(t+1) = \alpha N(t) \tag{4}$$

We determine the stable growth rate $\alpha - 1$ from historical data using the method proposed in Ekhosuehi and Osagiede [5].

Let the current staff-mix for the system be $\mathbf{x}(t) = [x_1(t) \quad \cdots \quad x_{\#(S)}(t)]'$ with $x_\nu(t) \geq 0$ and $\sum_{\nu=1}^{\#S} x_\nu(t) = 1$, so that $x_\nu(t)$ is the proportion of staff currently in category ν. The evolution of the structure $\mathbf{x}(t)$ at the next period is computed as $\mathbf{P'x}(t)$ and the $n-$step evolution is $\left(\mathbf{P'}\right)^n \mathbf{x}(t)$. However, $\mathbf{eP'x}(t) < 1$ and $\mathbf{e}\left(\mathbf{P'}\right)^n \mathbf{x}(t) < 1$, where $\mathbf{e}$ is a $1 \times \#(S)$ vector with each component equal to one. This is because $\mathbf{P}$ is sub-stochastic and $\mathbf{eP'} \leq \mathbf{e}$. The shortfall, $\mathbf{e} - \mathbf{eP'}$, is due to wastage in the system. So, $\mathbf{e}(\mathbf{I} - \mathbf{P'})$ is called the wastage vector, where $\mathbf{I}$ is an $\#(S) \times \#(S)$ identity matrix. The total wastage rate at the $n-$step is: $1 - \mathbf{e}(\mathbf{P'})^n \mathbf{x}(t)$.

The theoretical underpinning of our method is based on the following formula [1, 7]:

$$\mathbf{n}(t+1) = \mathbf{P'n}(t) + R(t+1)\boldsymbol{\rho} \tag{5}$$

where $R(t+1)$ is the total number of recruits in period t+1 and $\boldsymbol{\rho} = (\rho_\nu)$ is an $\#(S) \times 1$ vector of the recruitment policy with ρ_ν being the recruitment policy for the ν^{th} category, $\nu \in S$. The vector $\boldsymbol{\rho}$ is such that $\mathbf{e}\boldsymbol{\rho} = 1$. We assume that recruitment is done to replace wastage and to achieve the stable growth rate at the $n-$step [15] so that, for $n = 1$:

$$\frac{\mathbf{n}(t+1)}{N(t+1)} = \mathbf{P'} \frac{\mathbf{n}(t)}{\alpha N(t)} + \frac{\mathbf{e}(\mathbf{I} - \mathbf{P'})\mathbf{n}(t) + (\alpha - 1)N(t)}{\alpha N(t)} \boldsymbol{\rho} \tag{6}$$

Thus, we obtain

$$\mathbf{x}(t+1) = \alpha^{-1} [(\mathbf{P'})\mathbf{x}(t) + (\alpha - \mathbf{e}(\mathbf{P'})\mathbf{x}(t)) \boldsymbol{\rho}] \tag{7}$$

The aggregate-fractional flow balance equations are obtained by setting $\mathbf{x}(t+1) = \mathbf{D}$, so that

$$\alpha^{-1}[(\mathbf{P'})\mathbf{x}(t)+(\alpha-\mathbf{e}(\mathbf{P'})\mathbf{x}(t))\boldsymbol{\rho}]=\mathbf{D} \tag{8}$$

It is important to mention here that when $\mathbf{x}(t)=\mathbf{D}$ in Eq. (8), then the model is akin to the system in [1]. However, we solve the problem for the case when $\mathbf{x}(t)\neq\mathbf{D}$. At the $n-$step, Eq. (8) becomes

$$\alpha^{-1}[(\mathbf{P'})^n\mathbf{x}(t)+(\alpha-\mathbf{e}(\mathbf{P'})^n\mathbf{x}(t))\boldsymbol{\rho}]=\mathbf{D} \tag{9}$$

We use the constraint $\mathbf{e}\boldsymbol{\rho}=1$ to simplify Eq. (9) and get

$$[(\mathbf{P'})^n\mathbf{x}(t)\mathbf{e}+(\alpha-\mathbf{e}(\mathbf{P'})^n\mathbf{x}(t))\mathbf{I}]\alpha^{-1}\boldsymbol{\rho}=\mathbf{D} \tag{10}$$

It is worth noting that one way towards attaining the desired structure is for the administrative authority of the system to retrench all individuals and then recruit afresh such that $\boldsymbol{\rho}=\mathbf{D}$. This scenario is exhibited by the limiting solution to Eq. (10), i.e., $\lim_{n\to\infty}[(\mathbf{P'})^n\mathbf{x}(t)\mathbf{e}+(\alpha-\mathbf{e}(\mathbf{P'})^n\mathbf{x}(t))\mathbf{I}]\alpha^{-1}\boldsymbol{\rho}=\lim_{n\to\infty}\mathbf{D}$.

Since the matrix $\mathbf{P}$ is sub-stochastic, then $0\leq\lim_{n\to\infty}\left\|\mathbf{P}^n\right\|\leq\lim_{n\to\infty}a^n=0$, where $a=\sup\left\|\mathbf{P}\right\|$. This implies that $\lim_{n\to\infty}\mathbf{P'}=\mathbf{0}$. Therefore, $\boldsymbol{\rho}=\mathbf{D}$.

For $1\leq n<\infty$, we introduce an $\#(S)\times 1$ vector $\mathbf{T}=\alpha^{-1}\boldsymbol{\rho}$ and solve the following system of equations

$$[(\mathbf{P'})^n\mathbf{x}(t)\mathbf{e}+(\alpha-\mathbf{e}(\mathbf{P'})^n\mathbf{x}(t))\mathbf{I}]\mathbf{T}=\mathbf{D}, \tag{11}$$

$$\mathbf{e}\,\mathbf{T}=\alpha^{-n}.$$

Eq. (11) is the population-dynamic model. The solution to Eq. (11) is computed in the MATLAB environment by the following command:

$$\mathrm{T}=\mathrm{A}\setminus B \tag{12}$$

where

$$\mathrm{A}=\begin{bmatrix}[(\mathbf{P'})^n\mathbf{x}(t)\mathbf{e}+(\alpha-\mathbf{e}(\mathbf{P'})^n\mathbf{x}(t))\mathbf{I}]\\ \mathbf{e}\end{bmatrix},\ \mathrm{B}=\begin{bmatrix}\mathbf{D}\\ \alpha^{-1}\end{bmatrix}.$$

The solution, say $\mathbf{T}=(\zeta_\nu)$, contains entries which are unrestricted in sign. We interpret the value ζ_ν as follows: $\zeta_\nu>0$ indicates the necessity to recruit new entrants into category ν at the $n-$step, and $\zeta_\nu\leq 0$ indicates no recruitment into category ν at the $n-$step. The distribution of recruits at the $n-$step, $\mathbf{r}(t+n)$, is defined to be

$$\mathbf{r}(t+n)\overset{\Delta}{=}\left(\sum_{\nu=1}^{\#(S)}\tilde{\zeta}_\nu\right)^{-1}\tilde{\mathbf{T}} \tag{13}$$

where $\tilde{\mathbf{T}} = (\tilde{\zeta}_\nu)$ is an $\#(S)\times 1$ vector with $\tilde{\zeta}_\nu = \begin{cases} \zeta_\nu & \text{if } \zeta_\nu > 0 \\ 0 & \text{if } \zeta_\nu \le 0 \end{cases}$, according to our interpretation for ζ_ν. The symbol $\overset{\Delta}{=}$ means "is defined to be". The vector $\mathbf{r}(t+n)$ is a proxy for $\boldsymbol{\rho}$. The elements of $\mathbf{r}(t+n)$, denoted as $r_\nu(t+n)$, satisfy the relations $r_\nu(t+n) \ge 0$ and $\sum_{\nu=1}^{\#(S)} r_\nu(t+n) = 1$, for all $t+n$. $r_\nu(t+n) = 0$ means no recruitment should be made into category $\nu \in S$, and $r_\nu(t+n) > 0$ means that a proportion of $r_\nu(t+n)$ out of the total number of recruits as decided by the administrative authority should be recruited into category ν in period $t+n$.

The relative structure of the system, after recruitment at the $n-$step is obtained as:

$$\tilde{\mathbf{x}}(t+n) = \alpha^{-1}[\,(\mathbf{P}')^n \mathbf{x}(t)\mathbf{e} + (\,\alpha - \mathbf{e}(\mathbf{P}')^n \mathbf{x}(t)\,)\,\mathbf{I}\,]\mathbf{r}(t+n) \tag{14}$$

We use the Euclidean norm, ϕ_t, to measure the closeness between the desired structure and the augmented structure at period t, and then compare it to that of the initial structure. The norm ϕ_{t+n} is given as

$$\phi_{t+n} = \left\|\tilde{\mathbf{x}}(t+n) - \mathbf{D}\right\| \tag{15}$$

We provide a guided tour of the computational method by writing a computer program implementable in the MATLAB environment (see Appendix).

3. APPLICATION

We implement our procedure on a faculty academic staff structure [4]. The faculty consists of five departments, namely: Chemistry (CHM), Computer Science (CSC), Geology (GLY), Mathematics (MTH) and Physics (PHY). From the records, we obtain the following for each department:

CHM

Total academic staff stock: $N^{[CHM]}(0) = 25$, $N^{[CHM]}(1) = 25$, $N^{[CHM]}(2) = 25$, $N^{[CHM]}(3) = 23$, $N^{[CHM]}(4) = 22$, $N^{[CHM]}(5) = 21$, $N^{[CHM]}(6) = 25$.

The sub-stochastic matrix and the current staff-mix by rank

$$\mathbf{P}^{[CHM]} = \begin{bmatrix} \frac{40}{43} & \frac{2}{43} & 0 \\ 0 & \frac{35}{41} & \frac{5}{41} \\ 0 & 0 & \frac{52}{57} \end{bmatrix},\ \mathbf{x}^{[CHM]}(6) = \begin{bmatrix} \frac{9}{25} \\ \frac{4}{25} \\ \frac{12}{25} \end{bmatrix};$$

CSC

Total academic staff stock: $N^{[CSC]}(0)=16$, $N^{[CSC]}(1)=23$, $N^{[CSC]}(2)=24$, $N^{[CSC]}(3)=22$, $N^{[CSC]}(4)=22$, $N^{[CSC]}(5)=25$, $N^{[CSC]}(6)=27$.
The sub-stochastic matrix and the current staff-mix by rank

$$\mathbf{P}^{[CSC]}=\begin{bmatrix}\frac{98}{109} & \frac{9}{109} & 0\\ 0 & \frac{15}{16} & \frac{1}{16}\\ 0 & 0 & 1\end{bmatrix},\ \mathbf{x}^{[CSC]}(6)=\begin{bmatrix}\frac{16}{27}\\ \frac{9}{27}\\ \frac{2}{27}\end{bmatrix};$$

GLY

Total academic staff stock: $N^{[GLY]}(0)=14$, $N^{[GLY]}(1)=12$, $N^{[GLY]}(2)=12$, $N^{[GLY]}(3)=12$, $N^{[GLY]}(4)=13$, $N^{[GLY]}(5)=12$, $N^{[GLY]}(6)=13$.
The sub-stochastic matrix and the current staff-mix by rank

$$\mathbf{P}^{[GLY]}=\begin{bmatrix}\frac{23}{24} & \frac{1}{24} & 0\\ 0 & \frac{22}{25} & \frac{2}{25}\\ 0 & 0 & \frac{24}{26}\end{bmatrix},\ \mathbf{x}^{[GLY]}(6)=\begin{bmatrix}\frac{4}{13}\\ \frac{4}{13}\\ \frac{5}{13}\end{bmatrix};$$

MTH

Total academic staff stock: $N^{[MTH]}(0)=24$, $N^{[MTH]}(1)=26$, $N^{[MTH]}(2)=28$, $N^{[MTH]}(3)=28$, $N^{[MTH]}(4)=28$, $N^{[MTH]}(5)=29$, $N^{[MTH]}(6)=32$.
The sub-stochastic matrix and the current staff-mix by rank

$$\mathbf{P}^{[MTH]}=\begin{bmatrix}\frac{70}{83} & \frac{12}{83} & 0\\ 0 & \frac{37}{46} & \frac{9}{46}\\ 0 & 0 & \frac{33}{34}\end{bmatrix},\ \mathbf{x}^{[MTH]}(6)=\begin{bmatrix}\frac{14}{32}\\ \frac{6}{32}\\ \frac{12}{32}\end{bmatrix};$$

PHY

Total academic staff stock: $N^{[PHY]}(0)=14$, $N^{[PHY]}(1)=15$, $N^{[PHY]}(2)=11$, $N^{[PHY]}(3)=13$, $N^{[PHY]}(4)=13$, $N^{[PHY]}(5)=14$, $N^{[PHY]}(6)=15$.

The sub-stochastic matrix and the current staff-mix by rank

$$\mathbf{P}^{[PHY]}=\begin{bmatrix} \frac{35}{41} & \frac{4}{41} & 0 \\ 0 & \frac{23}{26} & \frac{2}{26} \\ 0 & 0 & \frac{10}{13} \end{bmatrix},\ \mathbf{x}^{[PHY]}(6)=\begin{bmatrix} \frac{8}{15} \\ \frac{4}{15} \\ \frac{3}{15} \end{bmatrix}.$$

The desired staff-mix by rank in line with the NUC specification is given by the vector $\mathbf{D}=[0.45 \quad 0.35 \quad 0.20]'$. An inspection of the transition matrices shows a high possibility of staying on in a category and consequently, a low progression rate between a category and the next higher category. None of the departmental staff-mix by rank is exact as the NUC specification. The monotone decreasing pattern from a category to the next higher category in the NUC staff-mix is not a feature of the staff-mix of CHM, GLY and MTH. The staff-mix of CHM is top-heavy, while that of CSC is bottom-heavy. It is almost uniform for GLY. PHY satisfies the NUC staff-mix in one category. We obtain the following results for each department for a $3-$step period.

$$\mathrm{A}^{\mathrm{CHM}}=\begin{bmatrix} 0.4074 & 0.2898 & 0.2898 \\ 0.1395 & 0.2571 & 0.1395 \\ 0.4156 & 0.4156 & 0.5332 \\ & & \\ 1 & 1 & 1 \end{bmatrix},\ \mathrm{B}^{\mathrm{CHM}}=\begin{bmatrix} 0.45 \\ 0.35 \\ 0.20 \\ \\ 0.9625^{-1} \end{bmatrix},\ \mathbf{r}^{\mathrm{CHM}}(9)=\begin{bmatrix} 0.4207 \\ 0.5793 \\ 0 \end{bmatrix};$$

$$\mathrm{A}^{\mathrm{CSC}}=\begin{bmatrix} 0.5193 & 0.4307 & 0.4307 \\ 0.3985 & 0.4870 & 0.3985 \\ 0.1414 & 0.1414 & 0.2300 \\ & & \\ 1 & 1 & 1 \end{bmatrix},\ \mathrm{B}^{\mathrm{CSC}}=\begin{bmatrix} 0.45 \\ 0.35 \\ 0.20 \\ \\ 1.0591^{-1} \end{bmatrix},\ \mathbf{r}^{\mathrm{CSC}}(9)=\begin{bmatrix} 0.3948 \\ 0 \\ 0.6052 \end{bmatrix};$$

$$\mathrm{A}^{\mathrm{GLY}}=\begin{bmatrix} 0.3774 & 0.2708 & 0.2708 \\ 0.2422 & 0.3488 & 0.2422 \\ 0.3654 & 0.3654 & 0.4719 \\ & & \\ 1 & 1 & 1 \end{bmatrix},\ \mathrm{B}^{\mathrm{GLY}}=\begin{bmatrix} 0.45 \\ 0.35 \\ 0.20 \\ \\ 0.9850^{-1} \end{bmatrix},\ \mathbf{r}^{\mathrm{GLY}}(9)=\begin{bmatrix} 0.6271 \\ 0.3729 \\ 0 \end{bmatrix};$$

$$\mathrm{A}^{\mathrm{MTH}} = \begin{bmatrix} 0.3453 & 0.2624 & 0.2624 \\ 0.2264 & 0.3093 & 0.2264 \\ 0.4622 & 0.4622 & 0.5451 \\ & & \\ 1 & 1 & 1 \end{bmatrix},\ \mathrm{B}^{\mathrm{MTH}} = \begin{bmatrix} 0.45 \\ 0.35 \\ 0.20 \\ \\ 1.0340^{-1} \end{bmatrix},\ \mathbf{r}^{\mathrm{MTH}}(9) = \begin{bmatrix} 0.5995 \\ 0.4005 \\ 0 \end{bmatrix};$$

$$\mathrm{A}^{\mathrm{PHY}} = \begin{bmatrix} 0.5468 & 0.3318 & 0.3318 \\ 0.3025 & 0.5175 & 0.3025 \\ 0.1432 & 0.1432 & 0.3582 \\ & & \\ 1 & 1 & 1 \end{bmatrix},\ \mathrm{B}^{\mathrm{PHY}} = \begin{bmatrix} 0.45 \\ 0.35 \\ 0.20 \\ \\ 0.9925^{-1} \end{bmatrix},\ \mathbf{r}^{\mathrm{PHY}}(9) = \begin{bmatrix} 0.5342 \\ 0.2086 \\ 0.2571 \end{bmatrix}.$$

The results show a contracting academic staff stock for CHM, GLY and PHY with $\alpha^{CHM} = 0.9625$, $\alpha^{GLY} = 0.9850$, and $\alpha^{PHY} = 0.9925$, respectively, while the stock for CSC and MTH is expanding with $\alpha^{CSC} = 1.0591$ and $\alpha^{MTH} = 1.0340$, respectively. At the $3-$step period, the results for CHM indicate that new entrants should be recruited into categories 1 and 2, but not into category 3. However, more recruits would be needed in category 2. In CSC, new entrants should be recruited into categories 1 and 3 with greater number of the recruits into category 3. There should not be recruitment into category 2. No recruitment should be made into category 3 in GLY, while new entrants are required in categories 1 and 2 with more recruits into category 1. A similar result for GLY holds for MTH. New entrants are required in all categories in PHY with the greatest number of the recruits into category 1.

The $3-$step augmented structure and the Euclidean norm, ϕ_9, as well as that of the initial structure, ϕ_6, for each department are obtained using equations (14) and (15) as:

$$\tilde{\mathbf{x}}^{[CHM]}(9) = \begin{bmatrix} 0.3525 \\ 0.2157 \\ 0.4318 \end{bmatrix},\ \phi_6 = 0.3501,\ \phi_9 = 0.2851;$$

$$\tilde{\mathbf{x}}^{[CSC]}(9) = \begin{bmatrix} 0.4397 \\ 0.3762 \\ 0.1841 \end{bmatrix},\ \phi_6 = 0.1910,\ \phi_9 = 0.0323;$$

$$\tilde{\mathbf{x}}^{[GLY]}(9) = \begin{bmatrix} 0.3428 \\ 0.2862 \\ 0.3710 \end{bmatrix},\ \phi_6 = 0.2369,\ \phi_9 = 0.2116;$$

$$\tilde{\mathbf{x}}^{[MTH]}(9) = \begin{bmatrix} 0.3019 \\ 0.2511 \\ 0.4470 \end{bmatrix},\ \phi_6 = 0.2391,\ \phi_9 = 0.3045;\ \text{and}$$

$$\tilde{\mathbf{x}}^{[PHY]}(9) = \begin{bmatrix} 0.4500 \\ 0.3500 \\ 0.2000 \end{bmatrix},\ \phi_6 = 0.1179,\ \phi_9 = 1.9429 \times 10^{-16}.$$

The Euclidean norm indicates that the augmented structure improves on the initial structure in four departments (CHM, CSC, GLY, and PHY), but not in MTH. In particular, the augmented structure for PHY equates the desired structure.

4. CONCLUSION

In this study, an attempt has been made to find a recruitment distribution that is capable of generating a desired structure after one or more steps in a manpower system where negative recruitment is not allowed. The task was to formulate a system of aggregate-fractional flow balance equations within a discrete-time Markov chain framework. Our model complements the existing model in the literature. One of the accomplishments of the study is the knack to figure out a way to avoid the possibility of obtaining negative entries in the recruitment distribution. We have illustrated the usefulness of our approach for a faculty in the University of Benin, Nigeria. The practical challenges of implementing the model in the university system may include bottlenecks such as the inadequacy of resources, the possibility of overstaffing, dearth of applicants with the requisite qualifications and cognate experience, etc. These challenges are grey areas for future research.

Acknowledgement: We thank the Editor and the reviewers for helpful comments which greatly improved the earlier manuscript.

REFERENCES

[1] Bartholomew, D. J., Forbes, A. F., and McClean, S. I., *Statistical techniques for manpower planning* (2nd ed.), John Wiley & Sons, Chichester, 1991.

[2] Chapman, S. J., *Essentials of MATLAB programming* (2nd ed.), Cengage Learning, USA, 2009.

[3] Davies, G. S., "Control of grade sizes in a partially stochastic Markov manpower model", *Journal of Applied Probability*, 19 (2) (1982) 439-443.

[4] Ekhosuehi, V. U., "Evaluation of career patterns of academic staff in a faculty in the University of Benin, Nigeria", *Ife Journal of Science*, 15 (1) (2013) 81-91.

[5] Ekhosuehi V. U. and Osagiede A. A., "A proposed Markov model for predicting the structure of a multi-echelon educational system in Nigeria", *Monografias Matematicas Garcia de Galdeano*, 38 (2013) 65-74.

[6] Feichtinger, G., "On the generalization of stable age distributions to Gani-type person-flow models", *Advances in Applied Probability*, 8 (3) (1976) 433 – 445.

[7] Gani, J., "Formulae for projecting enrolments and degrees awarded in universities", *Journal of the Royal Statistical Society*, Series A (General), 126 (3) (1963) 400-409.
[8] Gerontidis, I. I., "On certain aspects of non-homogeneous Markov systems in continuous time", *Journal of Applied Probability*, 27 (3) (1990) 530-544.
[9] Guerry, M. A., "Properties of calculated predictions of graded sizes and the associated integer valued vectors", *Journal of Applied Probability*, 34 (1) (1997) 94-100.
[10] Hahn, B. and Valentine, D., *Essential MATLAB for engineers and scientists* (4th ed.), Elsevier Ltd., UK, 2010.
[11] Haigh, J., "Stability of manpower systems". The *Journal of the Operational Research Society*, 43 (8) (1992) 753-764.
[12] Kipouridis, I. and Tsaklidis, G., "The size order of the state vector of discrete-time homogeneous Markov systems", *Journal of Applied Probability*, 38 (2) (2001) 357-368.
[13] Nicholls, M. G., "The use of Markov models as an aid to the evaluation, planning and benchmarking of doctoral programs", *Journal of the Operational Research Society*, 60 (2009) 1183-1190.
[14] Osasona, O., "Tools for academic planning", In: Uvah, I. I. (Editor), *Practical Guide on Academic Planning in Nigerian Universities: A Compendium of Academic Planning Tools,* (2012) 62-105.
[15] Tsaklidis, G. M., "The evolution of the attainable structures of a homogeneous Markov system with fixed size", *Journal of Applied Probability*, 31 (2) (1994) 348-361.
[16] Tsantas, N. and Vassiliou, P.-C. G., "The non-homogeneous Markov system in a stochastic environment", *Journal of Applied Probability*, 30 (2) (1993) 285-301.
[17] Vassiliou, P. –C. G., "A note on stability in Gani-type models in manpower systems", *Journal of the Operational Research Society*, 31 (1980) 953-954.
[18] Vassiliou, P. –C. G. and Georgiou, A. C., "Asymptotically attainable structures in nonhomogeneous Markov systems", *Operations Research*, 38 (3) (1990) 537-545.
[19] Vassiliou, P. –C. G. and Gerontidis, I., "Variances and Covariances of the grade sizes in manpower systems", *Journal of Applied Probability*, 22 (3) (1985) 583-597.
[20] Vassiliou, P. –C. G. and Tsaklidis, G., "The rate of convergence of the vector of variances and covariances in non-homogeneous Markov systems", *Journal of Applied Probability*, 26 (4) (1989) 776-783.
[21] Vassiliou, P. –C. G. and Tsantas, N., "Maintainability of structures in nonhomogeneous Markov systems under cyclic behaviour and input control", *SIAM Journal on Applied Mathematics,* 44 (5) (1984) 1014-1022.
[22] Vassiliou, P. –C. G., Georgiou, A. C. and Tsantas, N., "Control of asymptotic variability in non-homogeneous Markov systems", *Journal of Applied Probability*, 27 (4) (1990) 756-766.

APPENDIX

The MATLAB source codes for the computations

```
clc
disp('The Expansion/Contraction factor.')
R=[:]; %The vector of total stock over the years.
T=length(R); t=[1:T];

g=exp((12*t*(log(R))'-(6*(T+1)*sum(log(R))))/(T*(T^2-1))),

n=3;
```

```
P1=[:]; %Initial transition matrix.
P=P1^n,

x=[:]', %Current structure.


disp('Matrix of policy coefficients.')
e=ones(1,3);
A=[[P'*x*e+(g-e*P'*x)*eye(3)]; e],

D=[0.45 0.35 0.20 g^-1]';

disp('Recruitment policies.')
T=A\D,

if T(1,1)>0
    r1=T(1,1);
else
    r1=0;
end

if T(2,1)>0
    r2=T(2,1);
else
    r2=0;
end

if T(3,1)>0
    r3=T(3,1);
else
    r3=0;
end
rho=[r1 r2 r3]'/sum([r1 r2 r3]),
xcal=[P'*x*e+(g-e*P'*x)*eye(3)]*rho,
k=sum(xcal),
xpt=xcal/k,
EuD0=sqrt(sum((x-[0.45 0.35 0.20]').^2)),
EuD1=sqrt(sum((xpt-[0.45 0.35 0.20]').^2)),
```